GOVERNMENT OF INDIA
CENTRAL WATER COMMISSION
CENTRAL TRAINING UNIT

HYDROLOGY PROJECT

TRAINING OF TRAINERS
IN
HYDROMETRY

HOW TO ANALYSE STABILITY OF S-D RELATIONS

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1. MODULE CONTEXT

This module is a part of the 'Training in Hydrometry' for middle level engineers. This module is one of the two modules on 'Stage-Discharge Relations'. The two modules are :

Module	Code	Subject Contents
1. Understanding Stage - Discharge Relation		– Introduction to Stage - Discharge ratings, and Correlation and Regression – Classification of controls – Characteristics and Extrapolation of rating curves – Shifts in discharge ratings
2. How to analyse Stability of SD relation		– Fitting of curve for S-D relations – Testing the significance of curve fitting – Drawing of confidence limits – IS Code procedures

2. MODULE INFORMATION

Title	:	How to analyse Stability of S-D Relations
Target Group	:	Middle Level Engineers
Duration	:	90 minutes
Objectives	:	After training, the officers would be able to understand the concept of Stability of Stage-Discharge Relation and impart training to Supervisors and Junior Staff
Key Concepts	:	– Fitting S-D Relation – Tests for bias – Confidence band
Training methods	:	Lecture, discussions & questioning
Training aids	:	Overhead Projector, Transperancies, blackboard, Examples of Regression Analysis
Handout	:	Main text and Example

3. SESSION PLAN

Activity	Time
1. Introduction to Stability Concept	10 minutes
2. Explaining tests of significance	30 minutes
3. Discussions about tests	15 minutes
4. Explain the example	20 minutes
5. Questions and Answers	15 minutes
	90 minutes

INTRUCTORS NOTE

HOW TO ANALYSE STABILITY OF S-D RELATION

1.0 STABILITY OF STAGE-DISCHARGE RELATION

The stage-discharge relation curve, being a line of best fit, should be more accurate than any of the individual gaugings. However, any check discharge measurements conducted at the gauging station may not exactly fall on the already defined stage-discharge curve but may fall on either side of the curve. It is in this context that it becomes necessary to define the acceptable limits within which the observed discharge can deviate from the computed value using the stage- discharge relation. Though in some countries, the acceptable limit is defined to be +/- 5%, it is only empirical and is not supported by any scientific or statistical theories. Hence, it is necessary to introduce a concept of 'stability' of the stage- discharge relation. Using statistical analysis, it is possible to determine the 95% 'confidence limits' of the curve and a pair of curves can be drawn on either side of the stage-discharge curve to form a band. If 95% of the observations fall within this band, then the stage- discharge relation at that site can be considered stable. At the sites where the stage-discharge curve is stable, the frequency of the discharge observations can be reduced considerably and only stage measurements could be continued.

In India, at many of the hydrological observation sites, continuous daily discharge data of more than 20 to 25 years are available on the record. Using these data, the stability of stage-discharge relation at such sites can be examined. If any site is found to exhibit a stable stage- discharge relation, it should be possible to reduce the frequency of discharge observations at that site. However, gauge observations are required to be continued so that the corresponding values of discharge can be computed from the standardised stage- discharge relation.

2.0 STATISTICAL ANALYSIS AND INDIAN STANDARD RECOMMENDATIONS

There are two ways of defining the stage-discharge relation, one by fitting an equation using mathematical analysis as dealt in previous module and the other by fitting a smooth curve by eye. Whichever method is used to fit the curve, care should be taken to identify the change of controls and the curve shall be fit accordingly i.e. each part of the curve between the points of control shall be treated independently and the exercise carried out. The curve is to be subjected to various tests for goodness of fit and absence from bias with each part of the curve being tested separately. The Bureau of Indian Standards code IS: 2914-1964. 'Recommendations for Estimation of Discharges by Establishing Stage-Discharge Relations in Open Channels' has dealt the subject in great detail. The discussion that follows is largely based on the IS Code.

2.1 Testing of stage-discharge curves

The stage-discharge curves drawn/fit are to be tested for absence from bias, for goodness of fit, and for shifts in control. These tests are to be applied to the portions of the curves between the shifts in control, each individual portion being tested separately. As already discussed in previous module, it may not always be possible to fit a single mathematical equation for the entire range of stages and in many natural erodible channels (as is the case for most of the rivers in India), separate controls come into operation and cause composite curves with inflexions and discontinuties and in such a case a curve fit by eye may be best fit. The following tests are to be performed on the finalized stage-discharge curve.

2.1.1 Test -1

In a bias free curve drawn through 'N' observations, an equal number of observations are expected to be on either side of the curve. The actual number of points lying on either side should not deviate from N/2 by more than that can be explained by chance fluctuations in a binomially distributed variate with 1/2 as the probability of success. This is a very simple test and can be performed by counting the observed points falling on either side of the curve. If Q_O is the observed value and Q_E is the estimated value, then (Q_O - Q_E) should have an equal chance of being positive or negative. In other words, the probability of(Q_O - Q_E) being positive is 1/2. Hence assuming the successive signs to be independent of each other, the sequence of the differences may be considered as distributed according to the binomial law $(p+q)^N$, where N is number of observations, and p and q are the probabilities of occurrence of positive and negative values which are one-half each. For N greater than 30, a value of 't' (a statistical parameter) lower than 1.96 (say 2) indicates that the difference is not statistically significant at the 5% level.

Table 2.1 gives the computation details for performing the test.

TABLE 2.1
TEST 1 - Test for number of positive and negative deviations

S.No.	Particulars	Symbol	Rising Curve	Falling Curve
1.	Number of positive signs i.e points lying to the right side of the curve	n1		
2.	Total number of observations	N		
3.	Probability of a sign being +ve	p	1/2	1/2
4.	Probability of a sign being -ve	q	1/2	1/2
5.	Expected number of +ve signs	N.p		
6	Standard Deviations	$\sqrt{N.p.q}$		
7.	$\frac{\lvert n1 - N.P \rvert - 0.5 *}{N.p.q}$	t		

* --- continuity correction

If the values of 't' for both the curves fit for rising and falling stages are less than 1.96, then these curves are free from bias as judged by this test.

2.1.2 **Test - 2**

This test will not only ensure a balanced fit with regard to the deviations over different stages, but will also help in detecting changes in control at different stages . The discharge measurements shall be arranged in the ascending order of stage for this test. For a good graduation, a sign change in deviation is as likely as a non-change of sign giving rise to a binomial distribution with parameters (N-1) and 1/2. This test is based on number of changes of sign in the series of deviations (observed value minus estimated value). The signs of deviations of discharge measurements arranged in ascending order of stage are marked for example, as shown below :

+ - + + + - - + + …………….
1 1 0 0 1 0 1 0 …………….

Starting from the second number in the series, mark '0' if the sign agrees or '1' if it does not agree with the sign immediately preceding. If there are N deviations in the original series, there will be (N-1) numbers of the derived series 11001010.....If the observed values could be regarded as arising from random fluctuations from the estimated values from the curve, the probability of a change in the sign could be taken as one-half. It should be noted that this assumes that the estimated value is a median rather than mean. If N is fairly large (say 25 or more), a practical criterion may be obtained by assuming that the successive signs to be independent, (that is assuming as arising only from random fluctuations) so that number of 1's or 0's in the derived sequence of (N-1) members may be judged as a binomial variable with parameters (N-1) and 1/2.

Table 2.2 gives computational details for carrying out the test.

Table - 2.2
Test 2 - Test for Systematic trend in deviations

S.No.	Particulars	Symbol	Rising Curve	Falling Curve
1.	Number of Observations	N		
2.	Number of changes in sign	n		
3.	Probability of change in sign	p	1/2	1/2
4.	Probability of no change in sign	q	1/2	1/2
5.	Expected number of changes in sign	(N-1)p		
6.	$\frac{\lvert n - (N-P)p \rvert - 0.5 *}{\sqrt{(N-1)pq}}$	t		

If the value of 't' obtained is less than 1.96 for both the curves fit for rising and falling stages, then the test confirms that there is no systematic trend in the deviations.

2.1.3. Test - 3

The third test is designed to find out whether a particular stage-discharge curve, on an average, yields significant under-estimates or over-estimates as compared to the actual observations on which it is based. The percentage differences i.e.,

$$\frac{(Q_O - Q_E) \times 100}{Q_E} = p$$

are worked out and averaged. If there are N observations and if p1, p2.....pi....pn are the percentage differences, and if p is the average of pi's , the standard error S_E of p is given by

$$S_E = \sqrt{\frac{\sum (p-\bar{p})2}{N(N-1)}}$$

The average percentage $\bar{p}$ is tested against its standard error to see if it is significantly different from zero.

The percentage differences have been taken as they are rather independent of the discharge volume and are normally distributed about a zero mean value for an unbiased curve.

It is pertinent to note that the tests are to be carried out for rising and falling stages separately, if different curves are used to define the stage-discharge relationships. If, however, only a single curve is used for the purpose, then the tests are to be carried out for single curve assuming both the rising and falling stage observations to form homogeneous data, as illustrated in example.

2.2 Minimum number of observations

All the above tests shall be applied to portions of curves, each individual portion being tested for bias separately. Once the bias free curve is established, it may be checked if the number of observations chosen for establishing the curve are sufficient in number. Though this test need not be applied rigorously, it can be used to have an approximate idea of the minimum number of observations required for a good stage-discharge relation within the desired degree of confidence and the reliability of the estimate desired.

The discharge observations for a particular stage are likely to show wide variation due to random errors of measurements and various other factors. It is not unusual for individual points to vary by 20% or more from the mean stage-discharge relationship. Evidently, the greater the width of the scatter band, the greater should be the number of observations necessary to ensure that the mean relationship is determined with an acceptable degree of accuracy.

The variation of the percentage differences of the observed points from the curve of their mean relationship is measured by the Standard Deviation S_D. The Standard Deviation is the root mean square of the percentage differences.

The reliability of the mean relationship is measured by the Standard Error of the mean relationship S_E which is given by

$$S_E = \frac{S_D}{\sqrt{N}}$$ Where N is no. of observations.

The probability is approximately 20 to 1 that the shift of the apparent mean relationship (as determined from the observations) from the true relationship does not exceed $2S_E$. If the acceptable shift at a confidence level of 20 to 1 is set at p%, then $2S_E$ shall not exceed p.

But $S_E = S_D/ \sqrt{N}$,therefore, $2S_D/ \sqrt{N}$ shall not exceed p, from which it follows that N should not be less than

$$\{2SD/p\}^2$$

The Standard deviation shall be calculated separately for each range of stage having separate control. For each of these ranges, the N test should be applied separately to get the number of observations necessary to obtain a specified precision. An example given in IS:2914-1964 is reproduced below :

Table 2.3

Illustrative example for determination of number of observations required for establishing a reliable state- discharge relationship.

S.No.	Stage	Discharge Observed QO	Discharge Computed QC	Deviation (QO-QC)	Percent Deviation D	D^2
1	76.79	1682	1827	-145	-7.937	62.988
2	75.39	1644	1587	57	3.592	12.900
3	75.04	1598	1533	65	4.240	17.978
4	74.63	1390	1470	-80	-5.442	29.617
5	73.8	1382	1350	32	2.370	5.619
6	73.24	1353	1275	78	6.118	37.426
7	71.14	923	1014	-91	-8.974	80.539
8	70.59	1000	954	46	4.822	23.250
9	70.04	872	900	-28	-3.111	9.679
10	70.64	1002	960	42	4.375	19.141
11	71.69	1060	1084	-24	-2.214	4.902
12	72.04	1158	1123	35	3.117	9.714
13	69.94	912	888	24	2.703	7.305
14	69.61	810	855	-45	-5.263	27.701
15	68.40	802	745	57	7.651	58.538
16	68.09	651	721	-70	-9.709	94.260
17	67.31	687	661	26	3.933	15.472

18	66.61	620	615	5	0.813	0.661
19	66.1	577	581	-4	-0.688	0.474
20	65.41	559	548	11	2.007	4.029
21	64.91	464	521	-57	-10.940	119.695
22	64.41	478	496	-18	-3.629	13.170
23	63.91	449	472	-23	-4.873	23.745
24	63.16	435	434	1	0.230	0.053
25	62.81	426	418	8	1.914	3.663
26	62.21	384	389	-5	-1.285	1.652
27	61.96	416	378	38	10.053	101.061
Total					-6.128	785.232

Average $\overline{D}$ = -6.128 / 27 = - 0.22697

Sum of Deviation squares = D^2 = 785.232

$$(S_D)^2 = \frac{\sum D^2 - N(\overline{D})^2}{N-1}$$

$$= \frac{785.232 - 27 \times (-0.22697)^2}{26} = 30.148$$

Which implies S_D = 5.491

If the acceptable shift at a confidence level of 20 to 1 is set at 2% then the minimum number of observations necessary is

$$\{2SD/p\}^2$$

i.e.

$$\frac{4 \times (5.491)^2}{4} = 30.148 \text{ say } 30$$

In this case, the number of observations is "27" and hence, 3 more observations are required to satisfy the acceptable limit.

3.0 FIXING OF CONFIDENCE LIMITS

After the curve is fit and tested for absence from bias and minimum required number of observations are determined, it is now left to fix the 'confidence limits'. A pair of curves drawn to pass through points at a distance of $2S_E$ on either side of the stage- discharge curve are called the 95% confidence limits of the curve. These two curves define the limits within which the true value of discharge for a given stage should be in 95 cases out of 100.

The percentage Standard Error can be determined by the following formula

$$S_E = \sqrt{\sum \{Q_O - Q_E / Q_E * 100\}^2 / (N-2)}$$

* For (N-2) degrees of freedom

Where

N = Number of observations
Q_O = Observed discharge (Cumecs)
Q_E = Estimated discharge from the stage-discharge curve
S_E = Standard Error

The percentage standard error is then multiplies by `t' (1.96, for N>30 and 95% confidence level) and a pair of straight lines are drawn on the log-log plot of the stage- discharge curve and it is then verified by actual counting, if 95% of the observations are falling within the confidence limits. If so, the stage- discharge curve can be treated as stable and the stage- discharge relation so defined can be standardised for the gauging station, which shall then be checked periodically with the check gauging to detect the possible shifts in the rating in the future.

As long as the check gauging plot within the confidence limits, the established stage-discharge relation can be considered valid.

TEST ON CHECK GAUGINGS

The Students 't' test is used to decide whether the check gauging can be accepted as being part of the homogeneous sample of observations making up the stage-discharge curve. Such a test will indicate whether the stage-discharge relation of the station needs re- calibration or not.

The ratio of average deviation to the standard error of the difference of means should be less than 2.0 (for a 95% confidence) i.e.

$$t = \bar{d}/s \quad \text{should be less than 2.0}$$

$\bar{d}$ is the average of the percentage deviations
S is the standard error of the difference of the means which is given by

$$S = s\left[\frac{N + N1}{N \times N1}\right]^{1/2}$$

Where N is number of observations used to define S-D curve and

$$s = \left[\frac{\sum (D)^2 + \sum (d1 - \overline{d1})^2}{N + N1 - 2}\right]^{1/2}$$

$\sum(D)^2$	=	Sum of the squares of percentage deviation in the stage discharge curve.
d1	=	percentage deviation of the check gaugings
$\overline{d1}$	=	average of the percentage deviations of the check gaugings.

An illustrative example for carrying out the stability analysis, tests for absence of bias, Student's 't' test for check gauging is given in the following pages

EXAMPLE

Stability analysis of the stage-discharge curve of site AG000G7 Perur on river Godavari

E.1 Introduction

The hydrological observation station at Perur on Godavari river has a catchment area of 2,68,200 sq.km. with an average annual runoff of 265.53 mm and is located down stream of the confluence of the tributaries Pranahitha, Indravathi and Maner with main Godavari. The maximum estimated discharge at this station was 77,500 cumecs. The river with sandy bed is 1500m wide at this location and the banks are 10 m high made of black cotton soil. Daily gauge and discharge data is available from the year 1965.

Data considered

The gauge and discharge data of 11 years (for the years 1975 to 1985) were considered in this example. About 195 observations were selected covering the entire range of stages. While selecting the data, the following points have been kept in view :

1. All the observations at high stages, most of the medium stage observations and some of the low stage observations were considered.

2. Only observed values were considered and the estimated values have not been selected.

3. The points are so selected that the entire range of stages is covered uniformly.

E.2 Construction of stage-discharge curve

About 195 observed stage and discharge values covering the entire range of stages were selected for constructing the mean stage-discharge curve for the period 1975-85. Of the 195 samples selected, 98 were in rising stages and 97 were in falling stages. The stage- discharge relationship as manifested by these samples was plotted on a rectangular coordinate graph sheet taking the discharges on the abscissa and the stages on the ordinate. The plot is shown at Fig. E-1. As can be seen from the plot, the points in rising and falling stages are well distributed and do not form two distinct patterns. Thus, two different curves for rising and falling stages are not required. Hence, one single curve was fitted for both the rising and falling points put together. The sample points selected are as follows:

(i)	for stages from 70m to 81m			
RISING			FALLING	
STAGE	DISCHARGE		STAGE	DISCHARGE
(m)	(Cumecs)		(m)	(Cumecs)
70.010	192.0		70.965	389.5
70.120	75.0		72.030	733.0
71.033	690.0		72.043	903.0
71.588	518.5		72.118	1360.2
71.660	649.6		72.280	1136.1
72.170	1020.1		72.723	1512.8
72.620	1323.1		72.905	1555.1
72.805	1637.8		73.080	2044.0
73.035	737.3		73.170	2610.8
73.960	3303.0		73.235	1749.9
74.125	2673.0		73.300	2218.8
74.205	3807.5		73.320	2061.6
74.440	3418.4		73.400	2286.8
74.745	5198.4		73.419	2349.4
74.850	4239.4		73.420	1906.0
74.945	4543.9		73.603	2166.5
75.020	4630.9		73.700	2357.6
75.045	4493.0		73.705	2315.6
75.360	3458.7		73.870	1837.7
75.515	5278.9		73.985	2610.9
75.609	5890.7		73.990	3055.8
75.630	6353.7		74.050	3028.3
75.680	6665.7		74.110	3060.0
75.710	5790.2		74.120	3546.5
75.845	6627.1		74.169	3418.7
75.953	5985.7		74.380	3627.8
75.969	6590.8		74.445	4165.3
76.045	5169.0		74.460	3252.9
76.090	6349.5		74.643	4000.2
76.105	6522.8		74.770	3933.1
76.204	7074.0		75.070	5059.4
76.570	7855.1		75.070	5368.2
76.770	9022.4		75.170	4251.2
76.885	9207.5		75.190	4270.2
76.980	9720.9		75.200	3597.3
77.109	9188.7		75.255	4530.9
77.125	8951.9		75.290	6422.1
77.175	7923.5		75.370	4912.7

77.185	9897.1		75.380	5354.1
77.249	8700.8		75.600	5010.1
77.495	10542.9		75.610	5440.9
77.525	8950.7		75.700	5745.2
77.760	11315.1		75.845	4956.1
77.790	10129.6		76.050	6662.0
77.800	11716.7		76.080	7608.9
77.835	10442.7		76.390	7057.4
77.865	10986.0		76.449	7676.7
77.960	12458.5		76.490	9095.1
77.990	10469.6		76.699	7286.6
78.120	12985.8		76.710	7288.1
78.295	10661.5		76.765	8376.9
78.310	13077.4		76.775	6421.0
78.325	12929.3		76.950	8347.6
78.355	14325.2		77.005	8936.9
78.525	15376.2		77.260	7757.2
78.590	12834.8		77.560	10831.9
78.720	12434.3		77.569	10133.0
78.915	15136.3		77.750	11749.5
79.050	15399.8		77.780	11828.0
79.165	13938.1		77.815	11558.6
79.205	15857.7		77.835	10483.0
79.340	16364.7		77.985	9626.0
79.390	17560.3		78.050	12379.7
79.400	16440.0		78.270	13711.8
79.435	15409.1		78.290	13290.0
79.515	19147.9		78.330	14400.4
79.585	13959.2		78.380	12118.6
79.620	17078.9		78.443	14344.5
79.704	17846.9		78.685	15067.4
79.760	14434.1		78.855	16778.8
79.980	20455.5		79.020	15209.2
80.015	21086.5		79.240	17769.4
80.070	20068.0		79.470	15666.6
80.115	16503.4		79.510	14784.1
80.265	19089.9		79.605	13987.5
80.310	21472.8		79.720	19141.2
80.350	20955.9		79.750	19998.8
80.550	20970.6		79.755	18031.6
80.570	20947.9		79.850	17436.6
80.625	19930.1		79.940	18556.4
80.630	21121.3		80.070	20325.6
80.705	16558.9		80.195	19756.8
80.710	20430.4		80.235	21045.0
80.730	24365.0		80.450	22770.9

			80.585	20562.3
			80.885	23947.5
			80.980	22379.9
(ii)	for stages from 81m and above			
RISING			FALLING	
STAGE	DISCHARGE		STAGE	DISCHARGE
(m)	(Cumecs)		(m)	(Cumecs)
81.000	23418.2		81.130	25974.4
81.350	26069.3		81.394	26844.1
81.395	25704.8		81.470	26306.1
81.635	29028.0		81.665	24750.8
81.765	27829.5		81.685	29148.3
81.885	30053.4		81.795	29316.7
82.015	26479.6		81.869	29561.8
82.029	32181.0		80.073	29845.0
82.110	31292.0		82.295	33727.4
82.210	32811.4		83.475	43694.5
82.270	30499.9			
82.645	35306.3			
82.760	33697.3			
84.025	51496.0			

E.3 Fitting the Curve

One way of fitting the curve is by drawing a smooth curve by the judgment of eye. The curve so fitted should be tested for absence of bias as per IS Standards and if it is free from bias, it could be used for checking its stability.

Another away of fitting the curve is by means of a mathematical equation of the form

$Q = C (G-Go)^n$ where
Q = is discharge
G is Gauge height
Go is Gauge height for Zero discharge
C & n are constants

E.3.1 Estimating the Value of Go

Approximate value of Go is arrived at by using the formula

$$G0 = \frac{G1.G3-G2^2}{G1 + G3 -2G2}$$

Where G1.G2 & G3 are gauge heights corresponding to discharges Q1, Q2 & Q3 which are selected such that they are in a geometric progression (i.e. $Q2^2 = Q1 \times Q3$)

The values of Q are plotted against (G-Go) on log- log scale and Go (as calculated above) is adjusted slightly so that the points lie in a straight line. By trail and error, the corrected value of Go was obtained as 68.50 m. The log-log plot is shown at E-2

E-3.2 Fixing of different ranges for fitting the equation

A closer examination of the log-log plot and cross section reveals that the control of the stage-discharge curve has changed for stages above 81.0 . Hence, the entire range of stages has been split into two ranges (upto 81.0 m and above 81.0 m) and two separate equations were fit.

E-3.3 Fitting of mathematical equation

As described in section 4.0 of previous module using the regression analysis by the method of least squares i.e. by setting the sum of the squares of the deviation between log Q and log(G-Go) to a minimum, the following equations were arrived at

(a) Stages = 70m to 81m - $Q = 38.5344 (G-68.5)^{2.5330}$
(b) Stages = 81m to 85m - $Q = 4.8553 (G-68.5)^{3.3573}$

E-3.4 Testing for absence of bias:

The methodology described in Section 2.1 has been adopted to test the curves so fitted and the following results are obtained :

a) For range of stages from 70 to 81 m:

Test 1 : To check whether the curve is free from bias
(Ref:- Clause A-4.6 of IS-2914-1964)

S.No.	Particulars	Symbol	From Curve
1.	Number of positive signs i.e points lying to the right of the curve	n1	96
2.	Total number of observations	N	171
3.	Probability of a sign being +ve	p	1/2
4.	Probability of a sign being -ve	q	1/2
5.	Expected number of +ve signs	N.p	85.5
6	Standard Deviations	√N.p.q	6.54
7.	$\frac{\lvert n1 - N.P \rvert - 0.5 *}{N.p.q}$	t	1.53

*----- continuity correction

Since 't' is less than 2.0 it can be concluded that the curve is free from bias.

Test - 2 To check whether the curve is free from Systematic trend in deviations (Ref : Clause A-4.6 of IS-2914-1964)

S.No.	Particulars	Symbol	From Curve
1.	Number of Observations	N	171
2.	Number of changes in sign	n	85
3.	Probability of change in sign	p	1/2
4.	Probability of no change in sign	q	1/2
5.	Expected number of changes in sign	(N-1)p	85
6.	$\frac{\lvert n - (N-P)p \rvert - 0.5 *}{\sqrt{(N-1)pq}}$	t	0.08

Since 't' is less than 2.0, the curve is free from any systematic trend in deviation

TEST 3

Standard error of average % differences = 1.14
Average % difference = 1.11
Ratio = 1.11/1.14 = 0.974 which is permissible.

Hence test 3 is also okay.

b) For range of stages from 81 m and above :

Test 1 : To check whether the curve is free from bias
(Ref:- Clause A-4.6 of IS-2914-1964)

S.No.	Particulars	Symbol	From Curve
1.	Number of positive signs i.e points lying to the right of the curve	n1	15
2.	Total number of observations	N	24
3.	Probability of a sign being +ve	p	½
4.	Probability of a sign being -ve	q	½
5.	Expected number of +ve signs	N.p	12
6	Standard Deviations	$\sqrt{N.p.q}$	2.45
7.	$\frac{\mid n1 - N.P \mid - 0.5 *}{N.p.q}$	t	1.02

*----- continuity correction
Since 't' is less than 2.0 it can be concluded that the curve fit is free from bias

Test - 2 To check whether the curve is free from Systematic trend in deviations
(Ref : Clause A-4.6 of IS-2914-1964)

S.No.	Particulars	Symbol	From Curve
1.	Number of Observations	N	24
2.	Number of changes in sign	n	13
3.	Probability of change in sign	p	1/2
4.	Probability of no change in sign	q	1/2
5.	Expected number of changes in sign	(N-1)p	11.5
6.	$\frac{\mid n - (N-P)p \mid - 0.5 *}{\sqrt{(N-1)pq}}$	t	0.42

Since 't' is less than 2.0, the curve fit is free from any systematic trend in deviations

TEST 3

Standard error of average % differences = 1.05
Average % difference = 0.13
Ratio = 0.13/1.05 = 0.124 which is permissible.

Hence test 3 is also okay.

From the above it is seen that the curves fitted are free from bias and are satisfying all the requirements of an ideal well-represented curve.

E-3.5 Minimum number of observations required:

The procedure described in section 2.2 is followed and the minimum observations required is arrived at as under:

Stage range	Observations required	Observation considered
(i) 70m to 81m	221	171
(ii) 81m and above	26	24

It is seen that in the first range about 50 more sample points are to be included and in the second range about 2 more sample points are to be included to satisfy this requirement.

E-3.6 Testing for stability with in the 95% confidence limits:

Using the equation described in section 3.0, the percentage standard errors computed for the two ranges of stages are as follows :

(i) Stages from 70m to 81m -------------- 14.96%
(ii) Stages from 81m and above ------------ 5.27%

A pair of straight lines at a distance of $2S_E$ are drawn on the log-log plot as illustrated in Fig E-3 and the 'confidence limits' are fixed. By counting the number of points lying outside the confidence band, it is seen that nine observations are lying outside the confidence band i.e., 95.4% of the observations are lying within the band. Thus, it can be inferred that the stage- discharge relations arrived at for this hydrological observation station are stable and can be used as standard ratings for the station.

E-3.7 Students 't' test for check gaugings :

A few observed discharges have been selected and the equations developed have been checked for students 't' test as described in section 3.0 and the following are the results obtained :

(i) for stages from 70m to 81m:

STAGE (m)	DISCHARGE (Cumecs)
70.300	47.9
72.000	948.3
75.515	5278.9
79.340	16346.7

80.350	20955.9

STUDENTS "t" for check gaugings = 1.84

(ii) for stages from 81m and above :

STAGES(m)	DISCHARGE (Cumecs)
81.350	26069.3
82.110	31292.0

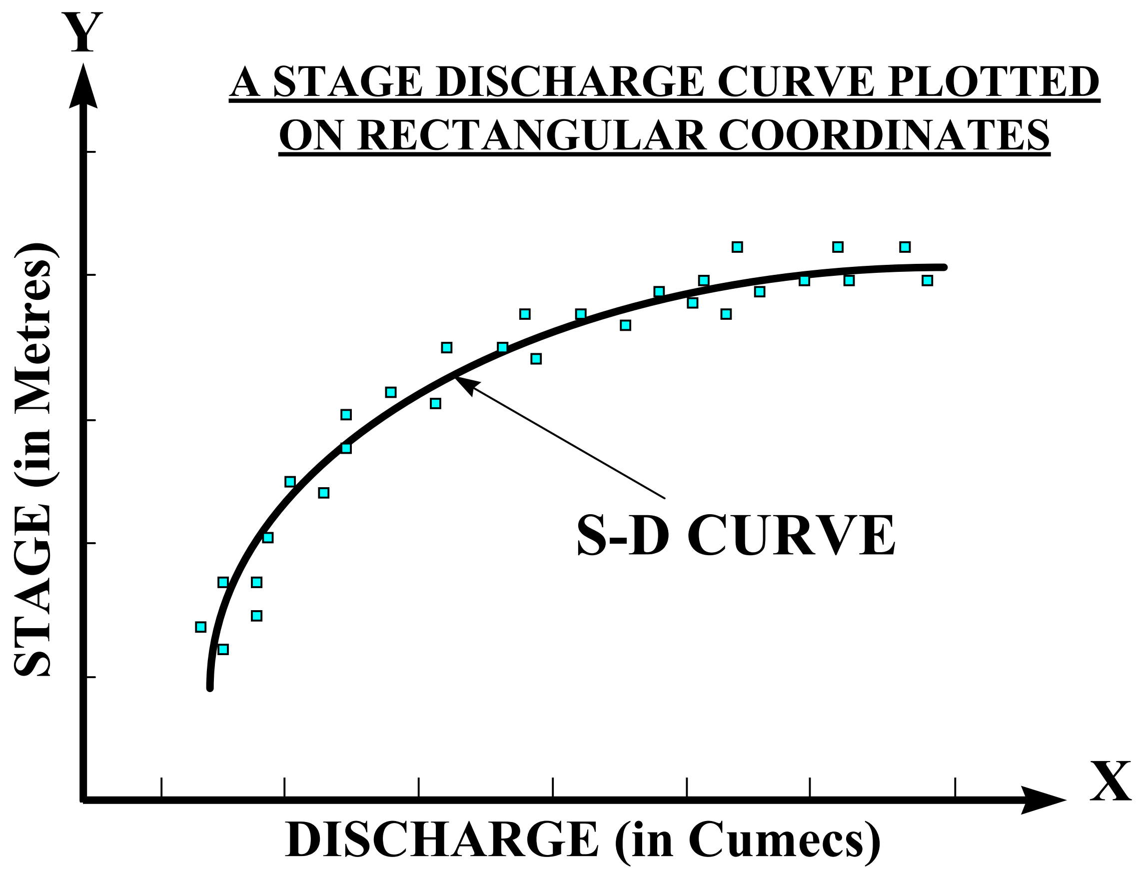
STUDENTS "t" for check gaugings = 0.30

Since t is less than 2.0, the test is okay in both cases.

OVERHEAD SHEETS

STEPS INVOLVED

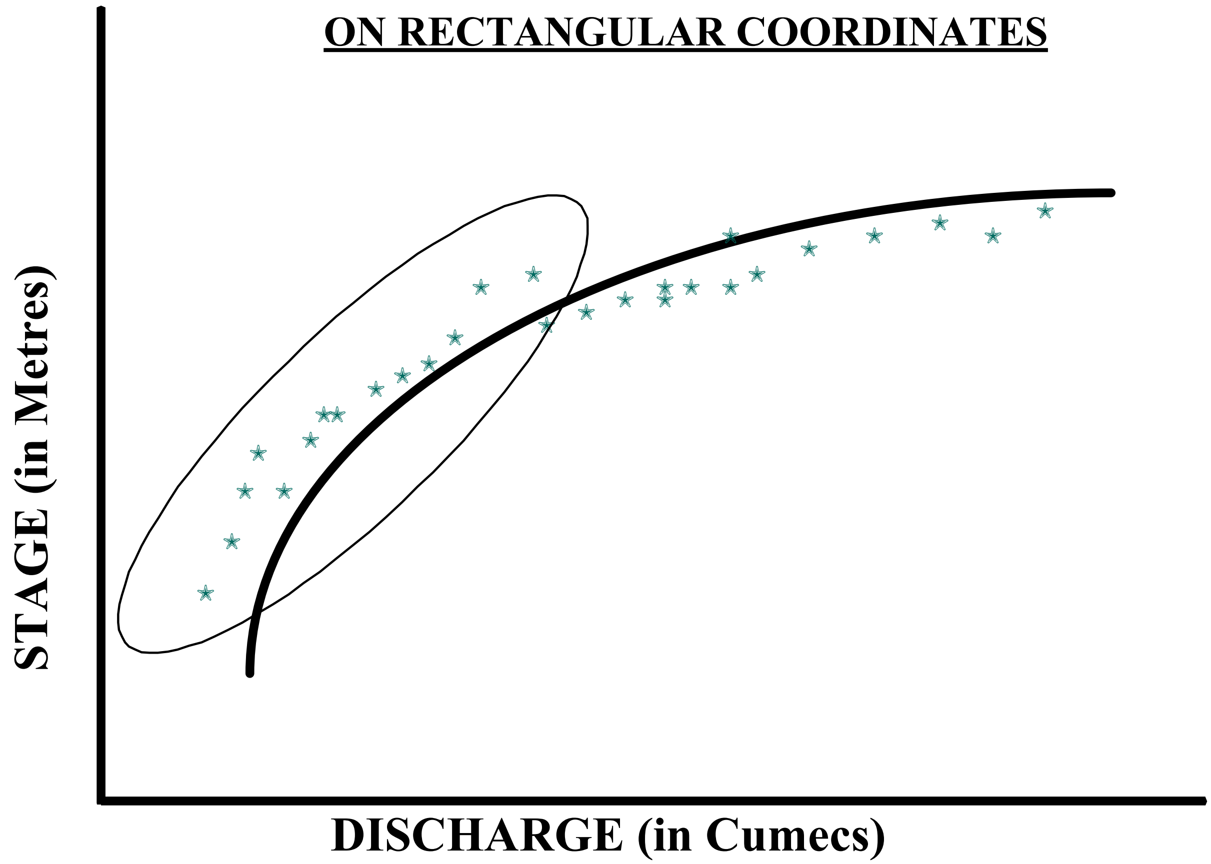
- ⇨ FINALISE THE CURVE
- ⇨ TEST FOR +VE AND -VE DEVIATIONS
- ⇨ TEST FOR SYSTEMATIC TREND IN DEVIATIONS
- ⇨ FIND MINIMUM NUMBER OF DATA POINTS
- ⇨ FIX THE CONFIDENCE LIMITS


Y
A STAGE DISCHARGE CURVE PLOTTED
ON RECTANGULAR COORDINATES
STAGE (in Metres)
S-D CURVE
X
DISCHARGE (in Cumecs)

TEST FOR +VE & -VE DEVIATIONS

Total no. of observartions	**N**
Number of positive signs i.e., points lying on the right side of the curve	**n1**
Probability of the sign being +ve	**p = 0.5**
Probability of sign being -ve	**q = 0.5**
Standard Deviation	**$\sqrt{N.p.q}$**
$t = \dfrac{\lvert n1 - N.p \rvert - 0.5}{\sqrt{N.p.q}}$	**if t < 1.96 then the curve is free from bias**

A STAGE DISCHARGE CURVE PLOTTED ON RECTANGULAR COORDINATES



TEST FOR SYSTEMATIC TREND IN DEVIATIONS

No. of observations	**N**
No. of sign changes	**n**
Probability of change in sign	**p = 0.5**
Probability of no change in sign	**q = 0.5**
Expected no. of changes in sign	**(N-1) p**

$$t = \frac{|\, n - (N-1)\, p \,| - 0.5}{\sqrt{(N-1)\, p\, q}}$$

If ‘t’ is < 1.96, the test is okay.

$$S_e = \left[\frac{\sum \left\{ \frac{Q_o - Q_e}{Q_e} \times 100 \right\}^2}{N-2} \right]^{1/2}$$

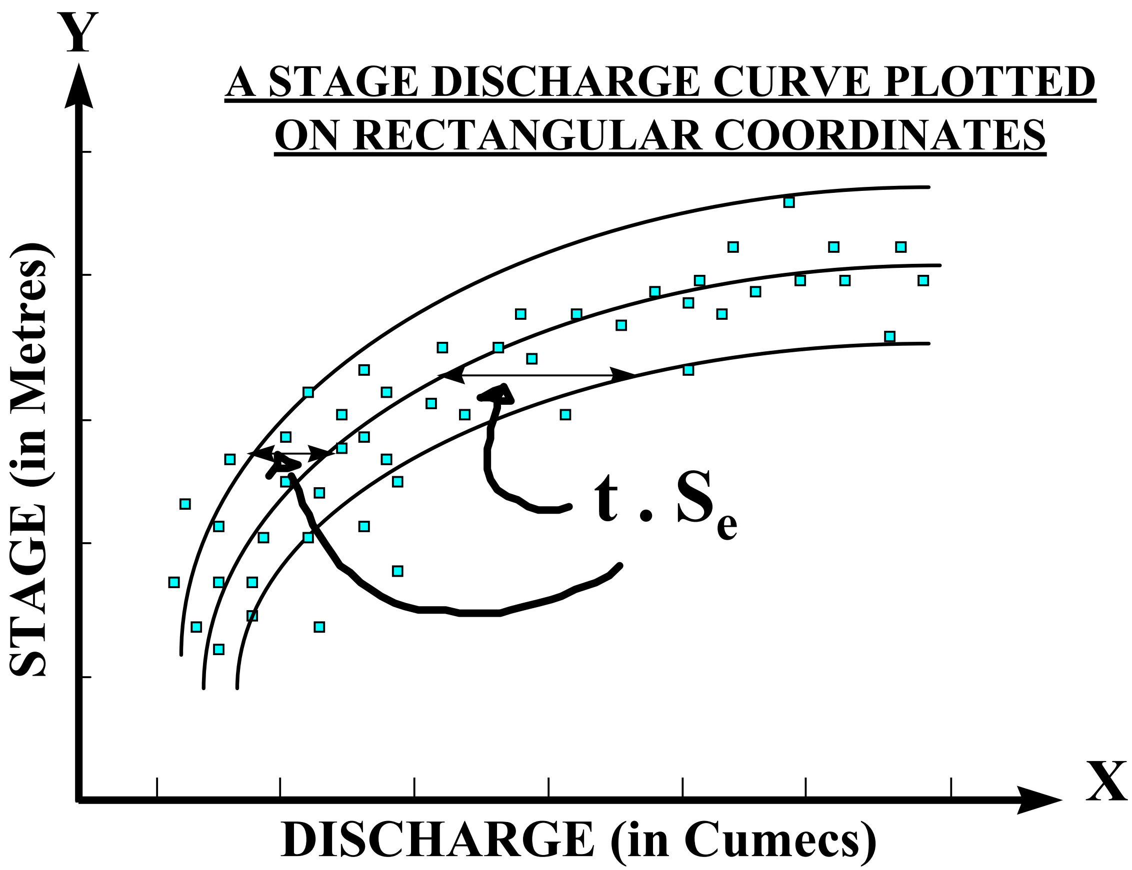
Q_o is the observed discharge

Q_e is the estimated discharge

N is the number of observations

The confidence band is ‘t’ times ‘ S_e’ wide on either side of the curve

t = 1.96 for N>30 and 95% confidence level


Y
A STAGE DISCHARGE CURVE PLOTTED
ON RECTANGULAR COORDINATES
t . Se
STAGE (in Metres)
X
DISCHARGE (in Cumecs)

SALIENT FEATURES

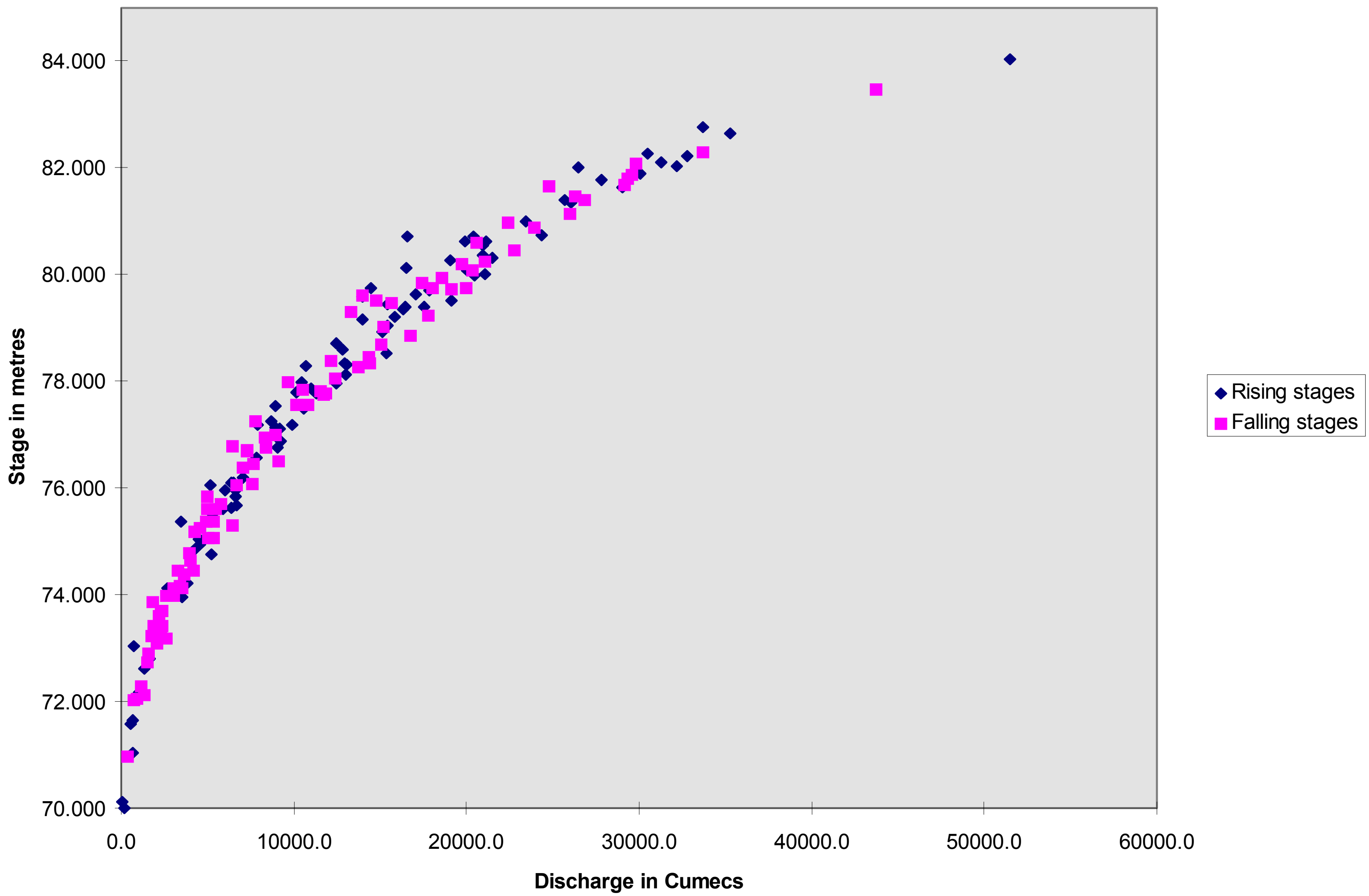
■ SITE	PERUR
■ BASIN	GODAVARI
■ C.A.	2,68,200 Sq Km
■ WIDTH	1500 m
■ MIN STAGE	70 m
■ MAX STAGE	84 m

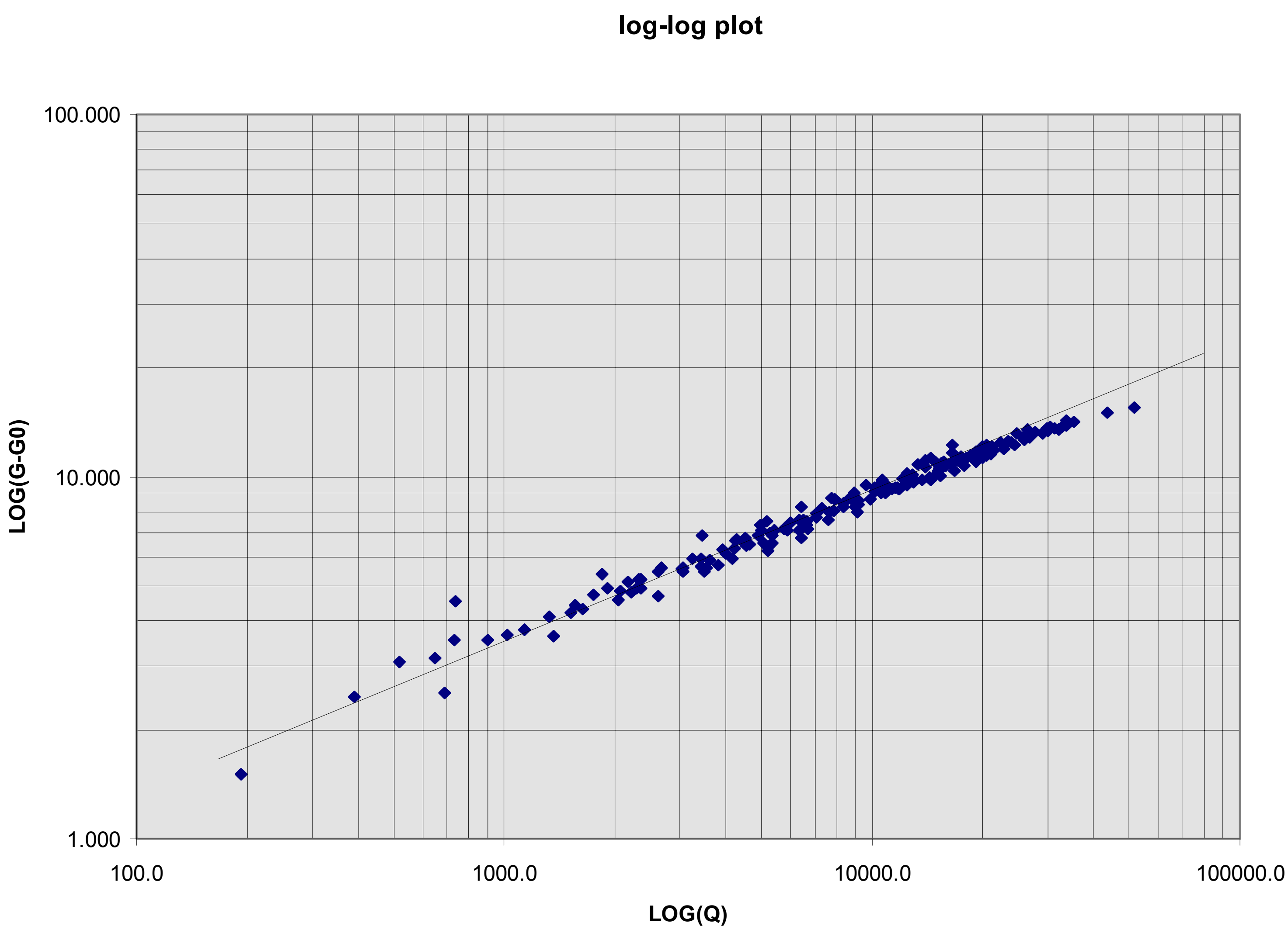
DATA SELECTED

ABOUT 195 OBSERVED STAGE & DISCHARGE VALUES WERE SELECTED FOR THE STUDY

THE DATA IS SPREAD OVER 11 YEARS (1975to 1985)

OUT OF 195 POINTS , 98 WERE IN RISING STAGES
97 WERE IN FALLING STAGES


84.000
82.000
80.000
78.000
76.000
74.000
72.000
70.000
Stage in metres
0.0
10000.0
20000.0
30000.0
40000.0
50000.0
60000.0
Discharge in Cumecs
Rising stages
Falling stages


log-log plot
100.000
10.000
1.000
100.0
1000.0
10000.0
100000.0
LOG(G-G0)
LOG(Q)

CHANGE OF CONTROL

A STUDY OF CROSS SECTION AND LOG-LOG PLOT OF DISCHARGE Vs (G-Go) REVEALED A CHANGE IN CONTROL AT A STAGE OF 81 m

THUS TWO CURVES WERE DERIVED

- **ONE FOR RANGE 1 ---- 70 to 81 m**
- **SECOND FOR RANGE 2 ---- 81 to 84 m**

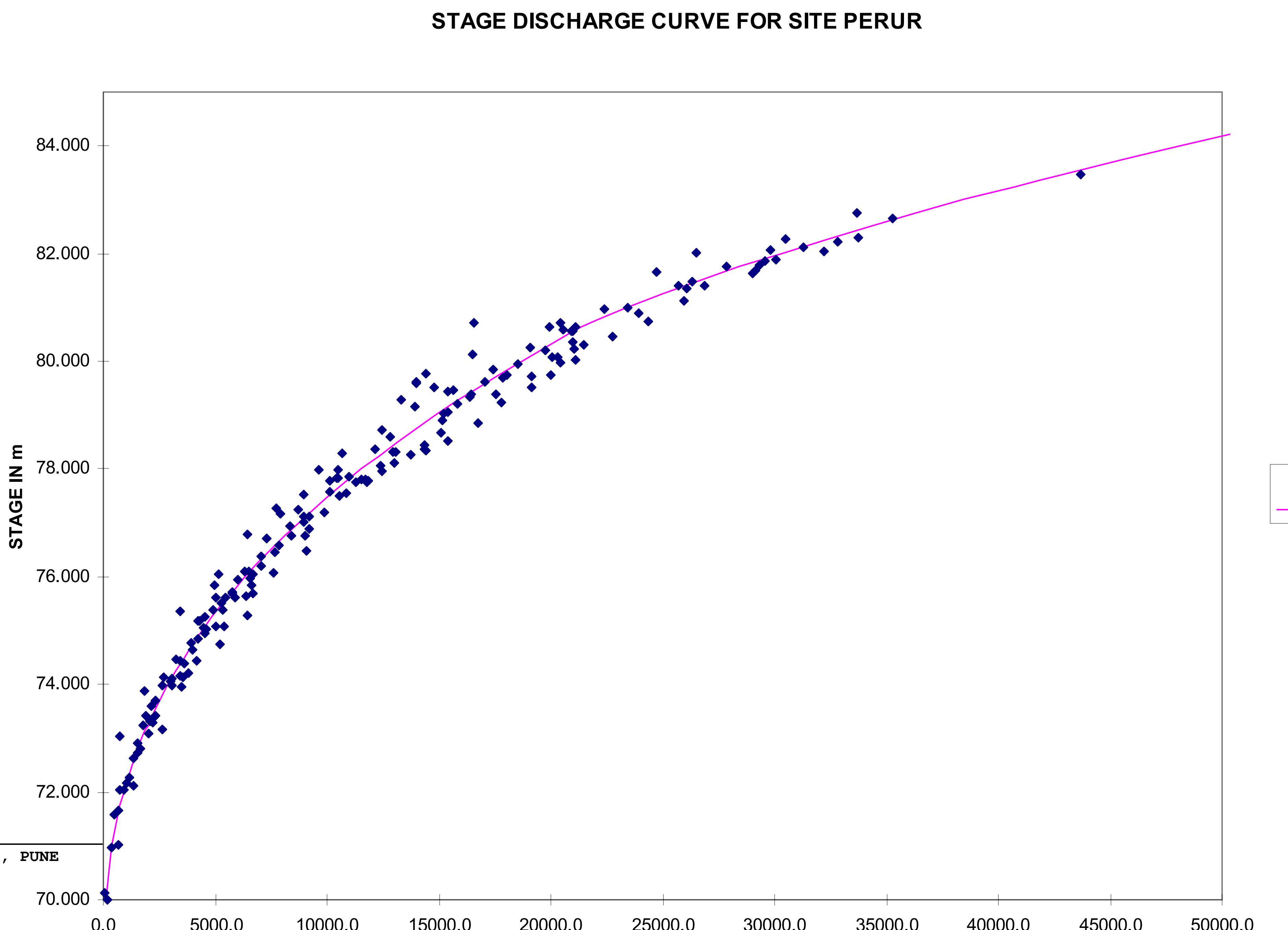
EQUATIONS DERIVED

FOR RANGE 1

$$Q = 38.5344\ (G-68.5)^{2.5330}$$

FOR RANGE 2

$$Q = 4.8553\ (G-68.5)^{3.3573}$$


STAGE DISCHARGE CURVE FOR SITE PERUR
STAGE IN m
84.000
82.000
80.000
78.000
76.000
74.000
72.000
70.000
0.0
5000.0
10000.0
15000.0
20000.0
25000.0
30000.0
35000.0
40000.0
45000.0
50000.0

TESTS FOR CHECKING BIAS

TEST 1

RANGE 1 - N = 171, n1 = 96 & t = 1.53

RANGE 2 - N = 24 , n1 = 15 & t = 1.02

TEST 2

RANGE 1 - N= 171, n = 85 & t = 0.08

RANGE 2 - N= 24, N= 13 & t=0.42

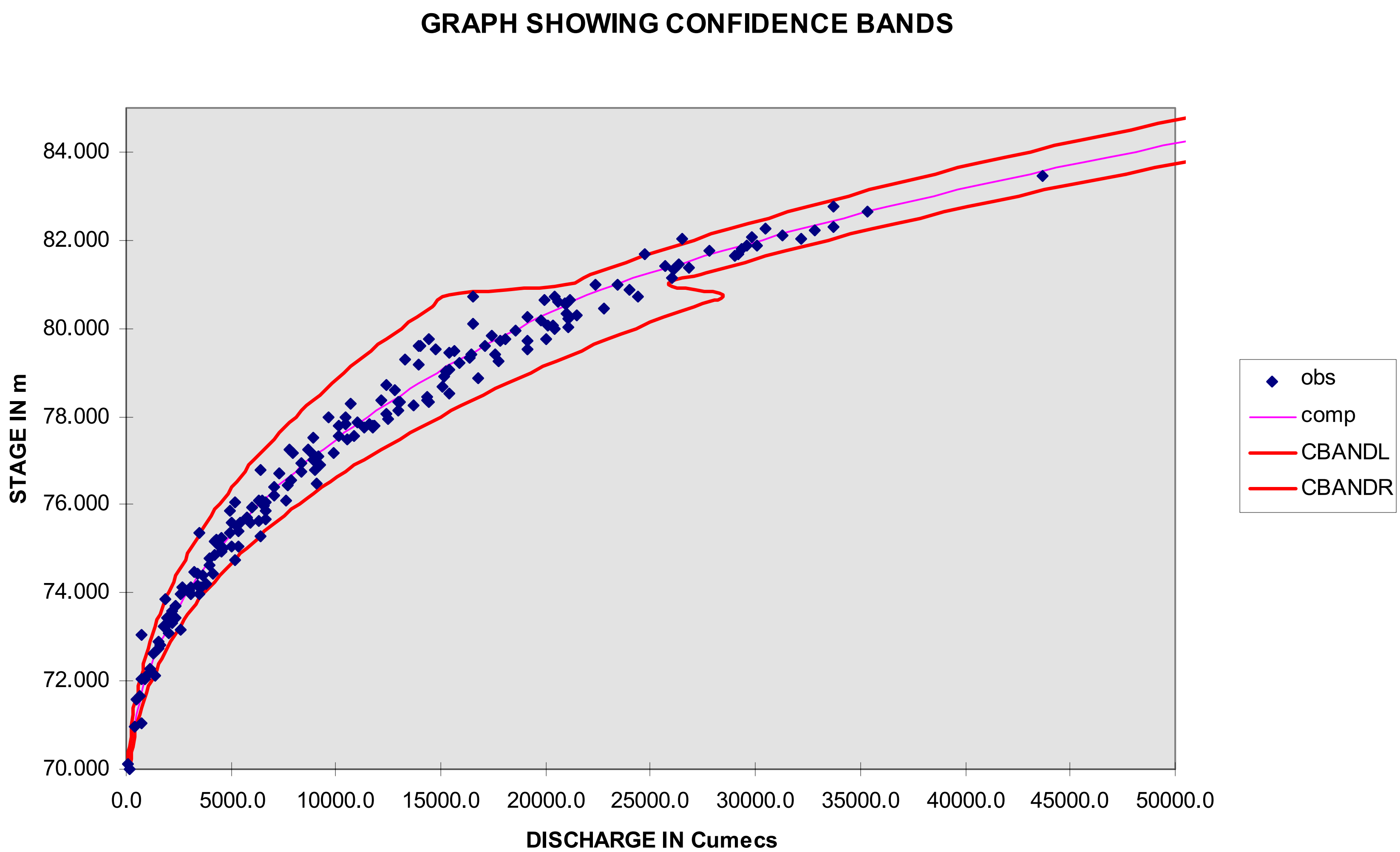
STANDARD ERROR

RANGE 1

STANDARD ERROR = 14.96 %

RANGE 2

STANDARD ERROR = 5.27%


GRAPH SHOWING CONFIDENCE BANDS
STAGE IN m
84.000
82.000
80.000
78.000
76.000
74.000
72.000
70.000
0.0
5000.0
10000.0
15000.0
20000.0
25000.0
30000.0
35000.0
40000.0
45000.0
50000.0
DISCHARGE IN Cumecs
obs
comp
CBANDL
CBANDR